Introduction to Tree Statics & Static Assessment

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Objectives:

1. Understand the load and stresses associated with simple tree/stem design and analysis.
2. Understand the stress-strain and load-displacement relationships for axial members.
3. Learn to calculate the stress, strain and displacement for beams under various loading conditions.
4. Learn to calculate stress, strain and displacement for torsional members, and to understand how power is transmitted through a tree.
5. Use mechanics of materials to analyze structures.
6. Try to relate to the real world in a simplified idealistic manner that gives usable results – tree inspection and assessment.
Content of presentation:

1. **Key Terms – Allowable Stresses and Allowable Loads**
2. **Current Tree Inspection Systems**
3. **Introduction to the Wood Science**
4. **Introduction to the Tree Biomechanics**
5. **Factors Affecting the Stability of the Tree**
3. Introduction to the Wood Science

1. Mechanical properties of wood

2. Stuttgart material properties of green wood

3. Compression failure

- The longitudinal axis \( L \) is parallel to the fiber (grain);
- the radial axis \( R \) is normal to the growth rings (perpendicular to the grain in the radial direction);
- and the tangential axis \( T \) is perpendicular to the grain but tangent to the growth rings.
Mechanical properties of wood

- Wood is **anisotropical** material due to composite structure.
- Mechanical behaviours are different among compression, tension, shear, bending and torsion, and also depend on the direction of loading (radial, tangential, longitudinal direction).
- Due to biological nature, wood is very **variable**.
- Properties are **influenced** by many factors (moisture, density, load duration, ……)

Wood = Fiber-reinforced composite
Important material parameters

- **E-modulus** - describes the **stiffness** of the material. It represents the stress necessary for the unit elongation of the material [MPa, kN/cm²].

- **Strength** - stress acting on the specimen [MPa, kN/cm²].

- **Strain** - relative **deformation** of the material (e.i. at the moment of failure, at proportional limit) [%,-].
3. Introduction to the Wood Science

**Strength of the wood**

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Moisture content</th>
<th>Specific gravity (kg.m-3)</th>
<th>Static bending (MPa)</th>
<th>Compression parallel to grain (MPa)</th>
<th>Tension parallel to grain (MPa)</th>
<th>Shear parallel to grain (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway spruce</td>
<td>Green 12</td>
<td>330</td>
<td>36</td>
<td>17</td>
<td>84</td>
<td>9</td>
</tr>
<tr>
<td><em>Picea abies</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>European beech</td>
<td>Green 12</td>
<td>550</td>
<td>65</td>
<td>28</td>
<td>130</td>
<td>9</td>
</tr>
<tr>
<td><em>Fagus sylvatica</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sycamore</td>
<td>Green 12</td>
<td>490</td>
<td>66</td>
<td>28</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td><em>Acer pseudoplatanus</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Introduction to the Wood Science

Stiffness of the wood

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Moisture content</th>
<th>Specific gravity</th>
<th>Modulus of elasticity</th>
<th>Modulus of rigidity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%)</td>
<td>(kg.m(^{-3}))</td>
<td>(MPa)</td>
<td>(MPa)</td>
</tr>
<tr>
<td><strong>Norway spruce</strong></td>
<td>Green</td>
<td>330</td>
<td>7 300</td>
<td>400</td>
</tr>
<tr>
<td><em>Picea abies</em></td>
<td>12</td>
<td>350</td>
<td>9 500</td>
<td>500</td>
</tr>
<tr>
<td><strong>European beech</strong></td>
<td>Green</td>
<td>550</td>
<td>9 800</td>
<td>800</td>
</tr>
<tr>
<td><em>Fagus sylvatica</em></td>
<td>12</td>
<td>600</td>
<td>12 600</td>
<td>1 100</td>
</tr>
<tr>
<td><strong>Sycamore</strong></td>
<td>Green</td>
<td>490</td>
<td>8 400</td>
<td>750</td>
</tr>
<tr>
<td><em>Acer pseudoplatanus</em></td>
<td>12</td>
<td>510</td>
<td>9 400</td>
<td>900</td>
</tr>
</tbody>
</table>
Stress – strain diagram (compression parallel to grain)

- **Ultimate strength**
- **Proportional limit**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{max}}$</td>
<td>64.3 MPa</td>
</tr>
<tr>
<td>E-Modulus</td>
<td>10649 MPa</td>
</tr>
<tr>
<td>$\mu_{\text{crit}}$</td>
<td>-</td>
</tr>
<tr>
<td>$\varepsilon_{\text{crit}}$</td>
<td>1.18 %</td>
</tr>
<tr>
<td>T-S</td>
<td>3.489e-003</td>
</tr>
<tr>
<td>Density</td>
<td>677.176 kg/m^3</td>
</tr>
</tbody>
</table>
3. Introduction to the Wood Science

![Diagram showing stress-strain relationship with key points labeled: Ultimate stress, Yield stress, Proportional limit, Perfect plasticity or yielding, Strain hardening, Necking, Fracture.]

- Engineering Stress/Strain
- True Stress/Strain
3. Introduction to the Wood Science

Stress–strain relationships for clear wood in compression and tension

![Diagram showing stress-strain relationship for clear wood in compression and tension.]
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Stress-strain diagrams for compression parallel to grain (Norway maple, *Acer platanoides*)

Stress-strain diagrams for green specimens

Stress-strain diagrams for green specimens (physiological active state)
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Stress strain diagrams for compression parallel to grain (Norway maple, *Acer platanoides*)

Stress-strain diagrams for dry specimens (w = 12%)
Stress strain diagrams for compression parallel to grain (static load capacity)

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Moist wood (MC>30%)
- Beech
- Oak
- Spruce

Dry wood (MC=12%)
- Beech
- Oak
- Spruce
3. Introduction to the Wood Science

**Influence of slope of grain**

<table>
<thead>
<tr>
<th>Maximum slope of grain in member</th>
<th>Modulus of rupture (%)</th>
<th>Impact bending (%)</th>
<th>Compression parallel to grain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight-grained</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1 in 25</td>
<td>96</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
<td>1 in 20</td>
<td>93</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>1 in 15</td>
<td>89</td>
<td>81</td>
<td>100</td>
</tr>
<tr>
<td>1 in 10</td>
<td>81</td>
<td>62</td>
<td>99</td>
</tr>
<tr>
<td>1 in 5</td>
<td>55</td>
<td>36</td>
<td>93</td>
</tr>
</tbody>
</table>

*Impact bending is height of drop causing complete failure (0.71-kg (50-lb) hammer); compression parallel to grain is maximum crushing strength.*

Strength of wood members with various grain slopes compared with strength of a straight-grained specimen.
<table>
<thead>
<tr>
<th>Common species names</th>
<th>Specific gravity</th>
<th>Modulus of elasticity (kN/cm²)</th>
<th>Deformation prop. limit (%)</th>
<th>Compression prop. limit (kN/cm²)</th>
<th>Drag coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>alder (Alnus)</td>
<td>0.86</td>
<td>800</td>
<td>0.25</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>ash (Fraxinus)</td>
<td>0.93</td>
<td>825</td>
<td>0.32</td>
<td>2.6</td>
<td>0.20</td>
</tr>
<tr>
<td>aspen (Populus)</td>
<td>0.76</td>
<td>680</td>
<td>0.24</td>
<td>1.6</td>
<td>0.20</td>
</tr>
<tr>
<td>basswood (Tilia)</td>
<td>0.84</td>
<td>700</td>
<td>0.25</td>
<td>1.75</td>
<td>0.25</td>
</tr>
<tr>
<td>beech (Fagus)</td>
<td>1.0</td>
<td>850</td>
<td>0.26</td>
<td>2.25</td>
<td>0.25 - 0.3</td>
</tr>
<tr>
<td>birch (Betula)</td>
<td>0.88</td>
<td>705</td>
<td>0.31</td>
<td>2.2</td>
<td>0.12</td>
</tr>
<tr>
<td>black locust (Robinia)</td>
<td>0.95</td>
<td>705</td>
<td>0.28</td>
<td>2.0</td>
<td>0.15 - 0.20</td>
</tr>
<tr>
<td>cedar (Chamaecyparis)</td>
<td>0.69</td>
<td>735</td>
<td>0.27</td>
<td>2.0</td>
<td>0.20</td>
</tr>
<tr>
<td>cedar (Juniperus)</td>
<td>0.75</td>
<td>765</td>
<td>0.20</td>
<td>1.5</td>
<td>0.15</td>
</tr>
<tr>
<td>douglas-fir (Pseudotsuga)</td>
<td>0.63</td>
<td>800</td>
<td>0.25</td>
<td>2.0</td>
<td>0.20</td>
</tr>
<tr>
<td>elm (Ulmus)</td>
<td>1.01</td>
<td>570</td>
<td>0.35</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>fir (Abies)</td>
<td>0.63</td>
<td>950</td>
<td>0.16</td>
<td>1.5</td>
<td>0.20</td>
</tr>
<tr>
<td>hornbeam (Carpinus)</td>
<td>0.99</td>
<td>880</td>
<td>0.18</td>
<td>1.6</td>
<td>0.25</td>
</tr>
<tr>
<td>horse chestnut (Aesculus)</td>
<td>0.92</td>
<td>525</td>
<td>0.27</td>
<td>1.4</td>
<td>0.35</td>
</tr>
<tr>
<td>chestnut (Castanea)</td>
<td>1.06</td>
<td>700</td>
<td>0.36</td>
<td>2.5</td>
<td>0.25</td>
</tr>
<tr>
<td>larch (Larix)</td>
<td>0.82</td>
<td>535</td>
<td>0.32</td>
<td>1.7</td>
<td>0.15</td>
</tr>
<tr>
<td>limetree (Tilia)</td>
<td>0.75</td>
<td>450</td>
<td>0.38</td>
<td>1.7</td>
<td>0.25</td>
</tr>
<tr>
<td>maple (Acer)</td>
<td>0.89</td>
<td>850</td>
<td>0.29</td>
<td>2.5</td>
<td>0.25</td>
</tr>
<tr>
<td>maple Norway (Acer)</td>
<td>0.92</td>
<td>700</td>
<td>0.36</td>
<td>2.55</td>
<td>0.25</td>
</tr>
<tr>
<td>oak english (Quercus)</td>
<td>1.1</td>
<td>790</td>
<td>0.35</td>
<td>2.8</td>
<td>0.25</td>
</tr>
<tr>
<td>oak pubescent (Quercus)</td>
<td>1.0</td>
<td>720</td>
<td>0.28</td>
<td>2.0</td>
<td>0.25</td>
</tr>
<tr>
<td>pine (Pinus)</td>
<td>0.82</td>
<td>700</td>
<td>0.24</td>
<td>1.7</td>
<td>0.15</td>
</tr>
<tr>
<td>poplar (Populus)</td>
<td>0.89</td>
<td>605</td>
<td>0.33</td>
<td>2.0</td>
<td>0.20 - 0.30</td>
</tr>
<tr>
<td>redwood (Sequoiadendron)</td>
<td>1.05</td>
<td>500</td>
<td>0.36</td>
<td>1.8</td>
<td>0.20</td>
</tr>
<tr>
<td>rowan tree (Sorbus)</td>
<td>1.07</td>
<td>600</td>
<td>0.27</td>
<td>1.6</td>
<td>0.25</td>
</tr>
<tr>
<td>spruce (Picea)</td>
<td>0.70</td>
<td>650</td>
<td>0.32</td>
<td>2.1</td>
<td>0.20</td>
</tr>
<tr>
<td>sycamore (Platanus)</td>
<td>0.99</td>
<td>625</td>
<td>0.43</td>
<td>2.7</td>
<td>0.25</td>
</tr>
<tr>
<td>tree-of-heaven (Ailanthus)</td>
<td>-</td>
<td>560</td>
<td>0.36</td>
<td>2.0</td>
<td>0.15</td>
</tr>
<tr>
<td>willow (Salix)</td>
<td>0.82</td>
<td>700</td>
<td>0.23</td>
<td>1.6</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Excessive compressive stresses along the grain that produce minute compression failures can be caused by excessive bending of standing trees from wind or snow; felling of trees across boulders, logs, or irregularities in the ground; or rough handling of logs or lumber.

The presence of compression failures may be indicated by fiber breakage on end grain. Since compression failures are often difficult to detect with the unaided eye, special efforts, including optimum lighting, may be required for detection. The most difficult cases are detected only by microscopic examination.

Even slight compression failures, visible only under a microscope, may seriously reduce strength and cause brittle fracture. Because of the low strength associated with compression failures, many safety codes require certain structural members, such as ladder rails and scaffold planks, to be entirely free of such failures.
1. Key Terms – Allowable Stresses and Allowable Loads

- **Structure** - any object that must support or transmit loads
- **Loads** - active forces that are applied to the structure by some external cause (i.e. wind)
- **Reactions** - passive forces that are induced at the supports of the structure (root area)
- **Properties** - types of members and their arrangement and dimensions, types of supports and their locations, materials used and their properties
- **Response** - how the structure will behave to loads (stress-strain diagram)
- **Stress** - force per unit area (normal stress, uniaxial stress)
- **Strain** - elongation per unit length (normal strain, uniaxial strain) (dimensionless)
1. Key Terms – Allowable Stresses and Allowable Loads

• Stiffness - the ability of the structure to resist changes in shape (e.g. stretching, bending, twisting)
• Strength - the ability of the structure to resist loads (i.e. compression members)

• Allowable load - permissible or safe load
• Allowable stress - the stress that must not be exceeded anywhere in the structure to satisfy the factor of safety

• Factor of safety - the ratio of actual strength to required strength (generally values from 1 to 10 are used) (structure will presumably fail for factor of safety less than 1)
Tree Statics & Static Assessment

(Part II)

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Content of presentation:

1. Introduction
2. Mechanical theory
3. Construction of the model
4. Growth stress
5. Factor of safety
6. Residual stem-wall
7. Case studies
Trees adapt their stem and root growth in response to the wind loading to which they are subjected in order to resist breakage or overturning.

By understanding the behaviour of trees in strong winds and the mechanisms of root anchorage it has become possible to develop mechanistic models that predict

1. the critical wind speeds for damage to occur and
2. how these are affected by the properties of the trees

Such an approach allows predictions of the impact of any arboricultural operations on tree stability and the design of strategies for reducing wind damage.
1. Introduction

- The distribution of longitudinal stresses in the stem due to its self-weight and several wind loading is calculated using the structural theory of a cantilever beam - **LOAD**

- The trunk of a tree has a specialised structure in order to support mechanical efforts, due to the self weight of the tree (crown and stem) and to the external loads (wind, snow) - **GEOMETRY**

- Wood structure, considered as a strengthening tissue, is supposed to be closely related to the stress level which affects it during the life of the tree - **MATERIAL**

- Although the relationship between wind loading and tree form has been studied, a detailed understanding of the effect of wind loading and tree weight on the internal wood structure and reactions has not been developed yet.
1. Introduction

Schematic diagram of mechanistic models

The basic structure of models is very similar and a general schematic relevant to models is shown in Fig.

The major differences lie in the method for calculating the values at each stage of the model.
2. Mechanical theory

**Schematic representation of the mechanic model.**

<table>
<thead>
<tr>
<th>Tree in consideration</th>
<th>Forces</th>
<th>Stresses</th>
<th>Sources</th>
<th>Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{w}$</td>
<td>$\sigma_{pa} = \frac{P_{w}}{A}$</td>
<td>Stem Compression (normal stress)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_{c}$</td>
<td>$\sigma_{pa} = \frac{P_{c}}{S_{t}}$</td>
<td>Crown Compression (normal stress)</td>
<td>Bending moment</td>
</tr>
<tr>
<td></td>
<td>$P_{h}$</td>
<td>$\sigma_{Mc} = \frac{M_{h}}{I_{h}}$</td>
<td>Wind Bending moment</td>
<td>Torsion moment</td>
</tr>
</tbody>
</table>
2. Mechanical theory

The main limitations of the adopted model are that it does not account for dynamic effects, and that growth stresses are not considered neither.

a) The stem of standing trees can be treated as an elastic cantilever beam, rigidly fixed on one side and free on the other. Its section varies with height, and this non-uniform taper can be described by a mathematical function.
b) The transverse section of the stem is considered circular, with an area $A$ and a section moduli $W$.
c) In order to calculate the self-weight of the tree, its canopy weight can be evaluated as a point vertical force applied in its centre of gravity.
d) In order to calculate the wind load, a horizontal point load applied also in the canopy centre of gravity can substitute it.
e) When bending, trees will usually fail on the compression side first, because wood is an extremely anisotropy material whose compression strength is about half the tensile strength. In the development of the method the most unfavourable case will always be considered, searching for the point where maximal compression occur.
2. Mechanical theory

Conclusion:

- The forces acting upon a tree are divided into
  1. the **horizontal force** due to the wind and
  2. the **vertical force** due to gravity, including the stem and crown weights and the weight of snow.

- Trees are assumed to deflect and/or to stretch to a point of no return when acted upon by wind of constant mean velocity and direction.
3. Construction of the model

1. Force due to wind

- There are a number of possible methods for calculating the wind loading on a tree. These include direct calculation from a knowledge of the drag coefficient and leaf area of the tree canopy (Jones, 1983), spectral methods using the approach pioneered by Davenport (1961) or an empirical approach using the measured drag of trees (Mayhead et al., 1975).

- The wind speed \( u \) over a forest's canopy is given by a logarithmic or power profile:

\[
v(z) = v(z_0) \left( \frac{z}{z_0} \right)^\alpha
\]

- The mean wind loading and gravity-based forces are calculated at each height in the canopy using a predicted wind profile and the vertical distribution of stem and crown weights.
3. Construction of the model

1. **Force due to wind**

The new Eurocode 1 includes four terrain categories with different roughness-parameters and in addition to that there are special windmaps based on different mean wind velocities for different locations:

![Profile of the mean wind velocity for different roughness-classes](image)

Profile of the mean wind velocity for different roughness-classes.
3. Construction of model

1. Force due to wind

- The total mean wind-induced force is the sum of the wind forces acting at each point on the stem and crown that is given (Jones, 1983; Peltola et al., 1999) at height \( z \) by:

\[
F_{\text{wind}} = \frac{1}{2} c_w \rho \, v_z^2 A
\]

where \( v \) is the mean wind speed,
\( A \) is the area of the stem and crown against which the wind acts,
\( c_w \) is the drag coefficient, and
\( \rho \) is the density of the air.
3. Construction of model

2. Forces due to crown and stem

The weight of the tree is divided into stem weight and canopy weight. As for the stem load, each section of the trunk is at any time supporting the weight of the portion of trunk

\[ F_{stem} = V_{stem} G_{stem} g \]

The canopy weight \( F_c \) is applied as a point load in the centre of gravity of the crown generating constant axial stresses like

\[ F_{crown} = m_{crown} g \]

Usually, the centre of gravity of the crown will be eccentric, and the distance to stem \( e \), and height \( h_{cg} \) can define its situation

\[ F_{crown} = m_{crown} \sin \left( \arctg \left( \frac{e}{h_{cg}} \right) \right) g \]
3. Construction of model

3. Compression stress

Axial stresses due to stem and crown mass loading vary along the stem with a maximum occurring at a position which depends on taper.

$$\sigma_{\text{tree}} = \frac{F_{\text{crown}} + F_{\text{stem}}}{A}$$

$$A = \frac{\pi}{4} D^2$$

$$A = \frac{\pi}{4} HB$$
3. Construction of model

4. Moments - bending

The bending of the stem is assumed to be directly proportional to the mean wind force acting on the crown centre and the height of center of gravity. The total maximum bending moment is at the base of the stem. Assuming that wind force is effective on the centre of gravity of the crown, the bending moment due to wind flow varies with the height of the cross-section considered.

\[ M_{\text{wind}} = F_{\text{wind}} h_{cg} \]

The effect of crown eccentricity was studied by Peltola and Kellomaki (1993): the eccentric load induces a bending moment which is constant along the stem. Once bending of a tree begins an additional force due to gravity is present and it produces bending moment

\[ M_{\text{crown}} = F_{\text{crown}} e \]
3. Construction of model

4. Moments - torque

The final moment – torsion moment – is produced by wind acting on eccentrically shifted center of crown gravity. The resultant load there is torque and stress acting on the tree there is shear.

\[ T_{\text{wind}} = F_{\text{wind}} e \]

Circular tubes are more efficient than solid bars in resisting torsional loads. Material near the outside surface carries most of the torsional load, so most of the material in a solid shaft is stressed significantly below the maximum shear stress.
3. Construction of model

5. Resistance to breakage

• The resistance to breakage is based on the assumption that the wind, crown and stem induced stress in the outer fibres of the tree stem is constant at all points between the base of the canopy.
• This allows the stress to be calculated only at given height (i.e. \( z = 1.3 \) m) and when this stress exceeds the distinct value – compression stress at proportional limit – the stem will break.
• The bending stress is given by the expressions:

\[
\sigma_{wind} = \frac{M_{wind}}{W} \quad \sigma_{crown} = \frac{M_{crown}}{W}
\]

• In a similar way, torsion moment results in torsion stress (torque):

\[
\tau_{wind} = \frac{T_{wind}}{W_T}
\]
3. Construction of model

5. Resistance to breakage

Both bending and torsion stresses are indirectly proportional to section moduli \( W \) given by equations:

\[
W = \frac{\pi D^3}{32}
\]

\[
W_x = \frac{\pi}{32} H^2 B
\]

\[
W_y = \frac{\pi}{32} HB^2
\]

\[
W_T = 2W
\]
3. Construction of model

5. Resistance to breakage

The most unfavourable case is to be considered, which means that wind flows in such a direction that compressive stresses due to wind add to the compressive stresses due to crown eccentricity.

Finally, the maximal compressive stress in the i-cross-section of the stem adopts the summation of all previously given expressions:

$$\Sigma\sigma = \frac{F_{crown}}{A} + \frac{F_{stem}}{W} + \frac{M_{crown}}{W} + \frac{M_{wind}}{W} + \frac{T_{wind}}{W_T}$$

$$\Sigma\sigma = \sigma_{tree} + \sigma_{crown} + \sigma_{wind} + \tau_{wind}$$
3. Construction of model

6. Resistance to overturning

• In a static system the uprooting forces, usually calculated as bending moments at the base of the stem, are treated as arising in two ways:

• Firstly, the force produced by wind action on the crown, simulated by pulling with a rope, causes defection of the stem. The leaning stem then assists in uprooting the tree because its centre of gravity moves over the hinge point in the root system (Ray and Nicoll, 1998).

• Thus, a second uprooting force is provided by the weight of the stem and crown. The uprooting moment is resisted by bending of the tree stem and various components of root anchorage:
  1. the weight of the root-soil plate,
  2. the strength of the windward roots,
  3. the strength of the root hinge and
  4. the soil strength at the base of the root-soil plate.
3. Construction of model

6. Resistance to overturning

- The resistance to overturning is based on tree pulling experiments.
- A tree will overturn if the total extreme bending moment due to the wind / load exceeds the support provided by the root-soil plate anchorage.

![Diagram showing the principles of overturning and anchorage with labels for wind, load, lever, inclinometer, and root spurs.](image-url)
3. Construction of model

6. Resistance to overturning

If the uprooting moment exceeds the resistive bending moment of the tree at a particular angle of deflection, the tree will deflect further. The tree will give way if the uprooting moment exceeds its maximum resistive bending moment, with the relative strengths of the stem and roots determining the mode of failure.

The evaluation of extremely tipped trees shows that the pattern is always the same: no further load increase is possible between 2° and 3° inclination. The Inclinometer method is based on this.
4. Factor of safety

Remember ....

**Factor of safety** - the ratio of actual strength to required strength (generally values from 1 to 10 are used) (structure will presumably fail for factor of safety less than 1)

\[
\text{factor of safety} = \frac{\sigma_{\text{compression}}}{\sigma_{\text{wind}} + \sigma_{\text{crown}} + \sigma_{\text{tree}}} \times 100
\]

\[
\text{factor of safety} = \frac{\tau_{\text{shear}}}{\tau_{\text{wind}}} \times 100
\]
5. Growth stress

- Growth stresses (Archer, 1986) have not been considered in the model, although they are very important in providing mechanical rigidity and in reducing the compression stresses on the bent tree.

- They act like a tensile pre-stress (Mossbrugger, 1990) which implies an enlargement of compressive strength because of a decrease of the effective stress supported.

- The value of the growth stresses is not usually over $5 \text{ MPa}$ (Fournier et al., 1990), which represents, in the case of the analyse the tree, that failure will occur at $u = 20 \text{ m/s}$ instead at $u = 16 \text{ m/s}$.

- Nevertheless, growth stresses could have some importance on the formation of heartwood, since usually stems are in tension in the outer rings and in compression in the inner ones, where heartwood is formed.
5. Growth stress

**Origin of growth stresses.**

To study the global stress distribution within a tree stem one needs to take into account, at each stage of tree life:

1. evolution of mass and geometry of the structure;
2. loading history, especially bending moments associated with leaning of the stem;
3. maturation stress occurrence in each new layer;
4. global mechanical equilibrium of the structure.

The respective history of growth and loading is a key point for this kind of analysis. The solving of such mechanical problem leads to **rather unexpected stress distributions** within a trunk or a branch, resulting from both the gravitational loading by the crown (the ‘support’ stress) and the maturation pre-stress. The summation of stresses resulting from support and maturation is classically called ‘growth stress’. 
6. Residual stem-wall

Remember ....

- Previous model considered the solid cross-section only and the calculation was performed under the wind load.
- However, there is constant omitted up to now – the STIFFNESS – the ability of tree to resist changes in shape (i.e. bending).
- E-modulus = ONLY material constant ever known (the criterion of the stiffness).
- If the wood is wood, then E-modulus must be constant because of the wood nature (chemical constitution and anatomical structure).
- Up to proportional limit

\[ \sigma = E\varepsilon_e \]

\( \varepsilon_e \) – only quantity to be easily measured
6. Residual stem-wall

- A winch system is used to pull the tree and the applied force needed to stretch fibers at the stem surface is measured at the given height of the stem – **PULLING TEST**

![Setup of Elastometer](image)

![Layout of tree-pulling system](image)

- Pulling (wintch) force and stretching of wood is recorded.
- Final QUESTIONS remain: If **E-modulus is material constant (TRUE)**, is the stretching proportional to the forces applied (TRUE/FALSE)? If not, what is probable cross-section of the stem at given height (CHANGES TO SECTION MODULUS)?
6. Residual stem-wall

And the solution is ....

1. Bending stress

\[ \sigma = \frac{M}{W} \]

\[ M = F_{\text{winch}} \cos \alpha (h_{\text{pull}} - h_e) \]

\[ W = \frac{\pi D^4 - d^4}{32 D} \]

2. Stress-strain relation

\[ \sigma = E \varepsilon_e \]

3. Insertion into equation (1)

\[ E \varepsilon_e = \frac{F_{\text{winch}} \cos \alpha (h_{\text{pull}} - h_e)}{\pi D^4 - d^4} \]

\[ \frac{32}{32} D \]

4. Final arrangement

\[ d = \left( D^4 - \frac{32 D F_{\text{winch}} \cos \alpha (h_{\text{pull}} - h_e)}{\pi E \varepsilon_e} \right)^{\frac{1}{4}} \]

where \( d \) – cavity diameter, \( D \) - stem diameter, \( E \) – modulus of elasticity, \( F \) – winch force, \( \varepsilon \) – measured stretching, \( h_{\text{pull}} \) – height of rope, \( h_e \) – position of Elastometer, \( \alpha \) – slope of rope
7. Case Studies

A. Stress distribution

• Several authors (Metzger, 1893; Petty, Swain, 1985; Mattheck, 1991; Wood, 1995) suggest that the stress should be constant in the stem periphery. As Mattheck (1991) demonstrates the relationship between stem diameter and height is defined by the postulate of constant axial stress on the stem surface of trees, these taper off towards the top in order to decrease the wind loads higher up; trees should maintain and restore the state of constant stress by permanent adaptation to the ever-changing external loads, and this leads to the concept of adaptive growth.

• On the other hand, it is possible that the strain, and not the stress, is constant in the stem periphery. It is easy to demonstrate (Wilson, Archer, 1979) that when an ideal beam of a shape following “the D³ law” (i.e. linear correlation between diameter cubed vs. height) is bent, the maximum strains along the external surface of the beam are constant along its whole length.

• Real trees do not often fit mathematical shape expressions closely, and there are differences in the type of parabolic followed by different species.
7. Case Studies

A. Stress distribution (*Pinus pinaster*)

Ezquerra, Gil (2001):

The following parameters were obtained directly by measuring the tree or its samples as described above:

\[ P_c = 99 \text{ kg} \quad h_t = 20.3 \text{ m} \quad h_g = 17.8 \text{ m} \quad e = 48 \text{ cm} \]

\[ \theta = 62.7^\circ \quad \rho = 1.23 \text{ g/cm}^3 \]

The most accurate expression (1) that represented the stem taper was \( r_i = 26.38 \cdot L_i^{0.3} \) (i.e. \( a = 26.38 \) and \( b = 0.5 \)), with \( r = 0.98 \).

An adequate wind velocity is chosen in order to determine the wind force: \( u = 15 \text{ m/s} \), that will generate stresses close to failure. For this velocity and for the considered species, we could take drag coefficient as \( C_d = 0.45 \). Thereby, the values needed to determine the wind effect are:

\[ u = 15 \text{ m/s} \quad C_d = 0.45 \quad A_e = 17.5 \text{ m}^2 \quad d = 1.226 \text{ kg/m}^3 \quad F_{we} = 1,086.2 \text{ N} \]
7. Case Studies

**A. Stress distribution** (*Pinus pinaster*)

![Graph showing stress distribution](image)
4. Introduction to the Tree Biomechanics

1. Wind-induced stresses in tree

2. Factors affecting forces acting on tree

3. Loads – axial loads and moments
4. Introduction to the Tree Biomechanics

- Failure occurs when the horizontal forces on a tree are transmitted down the trunk to create a stress that exceeds the resistance to breaking or turning of the root/soil system.
- As trees grow taller they can become increasingly prone to failure. For example, a force of 100 N applied at a height of 10 m creates a moment of 1000 Nm, but the same force at the 30 m height generates three times as much torque.
- Two horizontal forces contribute to the bending at each height increment.
  - **The first force** is a function of the effect of wind on the crown at given height.
  - **The second force** is a gravitational force that is contributed as the tree sways away from the vertical axis and/or eccentricity of center of gravity of the crown exists.
  - The gravitational force is relatively weak compared with the force of the wind on the crown until the tree starts to sway well away from the vertical axis. At a sway angle of 15-20°, the gravitational force can become a considerable proportion of the total horizontal force.
4. Introduction to the Tree Biomechanics

- The drag force on the crown is proportional to the area of branches and stems exposed to the wind, the drag coefficient of the foliage (i.e. how efficiently it intercepts wind), and the square of the wind speed (i.e. when the wind speed doubles, the drag force on the crown increases by a factor of four). Wind tunnel studies with whole trees have shown that the drag force is nearly proportional to the projected area of the canopy, drag coefficient, and wind speed.
- However, as wind speed increases, the canopy tends to bend and deflect and become more streamlined. Drag coefficients have been found to vary considerably between species.
- Taller individual trees growing within canopies that have uneven height or density distributions intercept more wind and therefore require stronger root anchorage to counter the increased drag force. The drag force of the wind on the crown results in branch and needle deflection.
- This force is transmitted to the stem, causing it to bend and sway.
1. Wind-induced stresses in tree

- The wind act in the area of the tree crown as in the sail of a ship.
- We can replace the acting forces in each one part of crown with the one solitary force acting in the centre of gravity of the crown.
- Than the calculation of the stresses and bending moments is enabled.

- Note that the force increase with the one half of sail area \((A)\), but with the square of the velocity \((v)\) !
- The \(C_x\) is the drag coefficient of the crown porosity, it depends on the species, on the wind velocity and other factors. Greek letter \(\rho\) denotes the density of the air \((1,2 \text{ kg.m}^{-3})\)

\[
F = 0.5 \rho C_x A v^2
\]
4. Introduction to the Tree Biomechanics

2. Factors affecting forces acting on tree

Factors affecting wind and gravitational forces acting on a tree.
4. Introduction to the Tree Biomechanics

2. Factors affecting forces acting on tree

Factors affecting the resistance to wind and gravitational forces acting on a tree.
4. Introduction to the Tree Biomechanics

2. Factors affecting forces acting on tree

<table>
<thead>
<tr>
<th>Independent Attribute</th>
<th>Windthrow Hazard</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Crown</td>
<td>Small Medium</td>
</tr>
<tr>
<td>Stem</td>
<td>Medium Taper</td>
</tr>
<tr>
<td>Roots</td>
<td>Moderately Deep</td>
</tr>
</tbody>
</table>

Crown, stem, and root attributes that affect the risk of failure.
3. Loads

The Loads – axial loads (normal and shear stresses) and moments (bending and torque):

- The main factor is the wind.
- Loads caused by the wind are much more higher then others.
- The „others“ include own weight of the tree, additional loads – the snow, the ice, the water (from rain), birds and other animals (for instance arborists …), and torque due to eccentricity of crown center of gravity.
3. Loads

Summary of mechanical stresses acting in trees

Figure 9: Main sources and effects of mechanical stresses acting in trees

Mattheck 1995
5. Factors Affecting the Stability of Trees

FACTORS AFFECTING WINDTHROW AND BREAKAGE OF TREE

- The factors that affect windthrow and breakage of trees are those that influence the effectiveness of root anchorage, the strength and aerodynamic properties of the tree, and the direction and characteristics of the wind within and above the stand.

- For simplicity these can be separated into
  1. individual tree characteristics,
  2. stand characteristics,
  3. root zone soil characteristics,
  4. topographic exposure characteristics,
  5. meteorological conditions.
5. Factors Affecting the Stability of Trees

1. Individual Tree Characteristics

At the individual tree level, the following characteristics affect tree stability:

- the height, diameter, and shape of the bole
- the crown class and size of crown
- the strength and elasticity of the bole, branches, and needles
- the rooting depth and area, size and number of roots, and whether or not adjacent tree root systems interlock.

2. Stand Level Characteristics

At the stand level, individual trees can be made more or less prone to windthrow through the effects of:

- stand height and density
- species composition
- silvicultural treatments (thinning, pruning, edge feathering, ripping, draining, etc.).
5. Factors Affecting the Stability of Trees

2. Stand Level Characteristics

A comparison of distributions of the relative windfirmness of individual trees comprising stands with different structural characteristics.
5. Factors Affecting the Stability of Trees

3. Soil Characteristics

Soil characteristics affect windthrow through the interaction of:
- depth
- drainage
- structure, density, texture, and the anchorage strength of the root system.

Root and soil factors affecting resistance to overturning.
5. Factors Affecting the Stability of Trees

4. Topographic Characteristics

*Topographic* characteristics affect windthrow by modifying:

- wind exposure
- wind direction, speed and turbulence.

<table>
<thead>
<tr>
<th>Roughness Type</th>
<th>Height of Boundary Layer [m]</th>
<th>Exponent $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat open country</td>
<td>270</td>
<td>1/7.0 = 0.14</td>
</tr>
<tr>
<td>rolling hills</td>
<td>390</td>
<td>1/3.5 = 0.28</td>
</tr>
<tr>
<td>inner city areas</td>
<td>510</td>
<td>1/2.5 = 0.40</td>
</tr>
</tbody>
</table>

Wind flow over a hill showing flow acceleration on the windward slope and turbulence (roller eddies) on the leeward slope.
4. Topographic Characteristics

The new Eurocode 1 includes four terrain categories with different roughness-parameters and in addition to that there are special windmaps based on different mean wind velocities for different roughness-classes.
4. Topographic Characteristics

The vertical profile of a graph of wind speed in the atmospheric boundary layer depends primarily on atmospheric stability, the roughness of terrain, the surfaces surrounding the building i.e., the ground and/or other buildings, and wind speed increases with increasing height above ground.

A wind velocity profile can be approximated either by a logarithmic equation or a power law expression:

\[ v(z) = v(z_0) \left( \frac{z}{z_0} \right)^\alpha \]

- \( v(z) \) = wind speed at height \( z \) [m/s],
- \( v(z_0) \) = wind speed at reference height \( z_0 \) [m/s],
- \( \alpha \) = exponent (0.16 – 0.40).
5. Factors Affecting the Stability of Trees

5. Meteorological Conditions

Meteorological conditions affect windthrow through the effects of:

- wind speed, gustiness, and storm duration
- soil moisture conditions
- snow and rain loading on the crown.

Wind velocity profile is determined by the roughness of the terrain. The value of the exponent $\alpha$ increases with increasing roughness of the solid boundary. For smaller areas of rough surfaces in smoother surroundings, such as a town located in flat, open country, the velocity profile described by the equation above is valid only for a limited height above the obstacles.
5. Factors Affecting the Stability of Trees

Conclusion:

- The concept of biomechanics refers to mechanical phenomena observed in a living plant, like a tree, that can be explained by the mere application of the usual analysis of structure and material mechanics.
- As an example, the global or local deformations of a tree submitted to sudden wind can be calculated by classical structure mechanics provided that sufficient information is given on:
  1. geometry,
  2. material properties and
  3. wind–structure interaction.
# Tree species susceptibility to storm and ice damage.

<table>
<thead>
<tr>
<th>Susceptible</th>
<th>Intermediate Resistance</th>
<th>Resistant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basswood</td>
<td>Ash – green &amp; white</td>
<td>Arborvitae</td>
</tr>
<tr>
<td>Birch – river &amp; paper</td>
<td>Buckeye</td>
<td>Baldeypress</td>
</tr>
<tr>
<td>Beech</td>
<td>Dogwood</td>
<td>Blackcypress</td>
</tr>
<tr>
<td>Boxelder</td>
<td>Eastern white pine</td>
<td>Blackgum</td>
</tr>
<tr>
<td>Black cherry</td>
<td>Eastern reedcedar</td>
<td>Black walnut</td>
</tr>
<tr>
<td>Black locust</td>
<td>Hawthorn</td>
<td>Eastern hemlock</td>
</tr>
<tr>
<td>Bradford pear</td>
<td>Hickories</td>
<td>Ginkgo</td>
</tr>
<tr>
<td>Crabapple</td>
<td>Loblolly pine</td>
<td>Holly</td>
</tr>
<tr>
<td>Elm – Chinese &amp; Siberian</td>
<td>Maple – red &amp; sugar</td>
<td>Linden</td>
</tr>
<tr>
<td>Hackberry</td>
<td>Persimmon</td>
<td>Kentucky coffeetree</td>
</tr>
<tr>
<td>Magnolia</td>
<td>Red oaks</td>
<td>Serviceberry</td>
</tr>
<tr>
<td>Silver maple</td>
<td>Sassafras</td>
<td>Shortleaf pine</td>
</tr>
<tr>
<td>Virginia pine</td>
<td>Sourwood</td>
<td>Sweetgum</td>
</tr>
<tr>
<td>Willow</td>
<td></td>
<td>White oaks</td>
</tr>
</tbody>
</table>

*Adapted from:* Hauer et al. 1994 and Barry et al. 1982.